STRESS LAYER ATTACHED TO HALF PLANES

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1. It was shown in [1] that a stress layer subjected to compressive stresses is formed along the weld line behind the point of contact during the collision of plates in the explosive welding mode. A hypothesis was advanced that the presence of the stress layer is the reason for wave formation, which occurs in some collision modes. It was established in experiments on explosive welding that the wave formation occurs at some time afterwards and not at the time of collision; i.e., an initial perturbation is necessary for the beginning of the wave formation process [2]. Such a perturbation might yield a rarefaction wave arriving from the free surface of the plate being thrust.

The question of the possibility of a wave-formed break in the stress layer $S=\{x, y \mid -\infty < x < +\infty, -h \le y \le h\}$ under the condition that this layer is compressed by stresses close to the yield point of the material but not exceeding it, is investigated in this paper. The case when the layer is in the plastic state was studied in [1]. Let the stress layer be bent wavily under the effect of a small perturbation. Since a layer compressed by stresses close to the material yield point is considered, then it is natural to assume that plastic hinges are formed at the vertices of the bending. Henceforth, the whole stress layer is considered as a "pin–joint system" connected by hinges. We shall assume that the rods are absolutely elastic and that the whole "pin–joint system" can leave the state of rectilinear equilibrium under the effect of a small perturbation. The compressive stresses [1]

$$\sigma_{11}^0 = -k \; (k > 0),$$

where

$$k = \frac{4}{3\pi\tau_s}\rho_0 HU \sin(\gamma/2)\left(\frac{c_0^2 - 2b_0^2}{c_0^2 - b_0^2}\right); \tag{1.1}$$

act in the layer S, where c_0, b_0 are the longitudinal and transverse speeds of sound in the material, ρ_0 is the density of the material, U is the velocity of the contact point, γ is the collision angle, H is the height of the plate, τ_s is the characteristic relaxation time of the tangential stresses in the plastic deformation zone located in the neighborhood of the contact point.

A vertically directed force $f = 2kh\sin\beta$, where β is the angle being formed between the rods, will act at the vertices of the break because the pin–joint system leaves the rectilinear equilibrium state. The foundations of the colliding plates will hinder the break of the pin–joint system. The height of the stress layer [1] is small compared to the height of the plate, and we shall hence consider the pin–joint system to be fastened to the half planes $P_1=\{x, y \mid -\infty < x < \infty, -\infty \le y < -h\}$, $P_2=\{x, y \mid -\infty < x < \infty, h < y < \infty\}$. The stress layer (the "pin–joint system") leaves the rectilinear equilibrium state under the effect of a small perturbation if the reactive force 2P, occurring because of bending of the foundation of the two half planes, is less than the force f which occurs at the vertices of the break in the "pin–joint system" (Fig. 1).

2. Let the stress state in the (x, y) plane be described by a system of equations of nonlinear elasticity theory (these equations are one of the forms of writing the equations in [3])

$$\begin{gathered}
\rho d\bar{u}/dt - \partial\sigma_{11}/\partial x - \partial\sigma_{12}/\partial y = 0, \\
\rho d\bar{v}/dt - \partial\sigma_{12}/\partial x - \partial\sigma_{22}/\partial y = 0, \\
\frac{d\ln k_1}{dt} - \cos^2\varphi\frac{\partial\bar{u}}{\partial x} - \sin\varphi\cos\varphi\left(\frac{\partial\bar{u}}{\partial y} + \frac{\partial\bar{v}}{\partial x}\right) - \sin^2\varphi\frac{\partial\bar{v}}{\partial y} = 0, \\
\frac{d\ln k_2}{dt} - \sin^2\varphi\frac{\partial\bar{u}}{\partial x} + \sin\varphi\cos\varphi\left(\frac{\partial\bar{u}}{\partial y} + \frac{\partial\bar{v}}{\partial x}\right) - \cos^2\varphi\frac{\partial\bar{v}}{\partial y} = 0, \\
\frac{d\ln k_3}{dt} = 0,
\end{gathered} \tag{2.1}$$

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$$2\left(k_1^2-k_2^2\right)\frac{d\varphi}{dt}+\left(k_1^2-k_2^2\right)\left(\frac{\partial\bar{u}}{\partial y}-\frac{\partial\bar{v}}{\partial x}\right)-$$
$$-\left(k_1^2+k_2^2\right)\left[\cos 2\varphi\left(\frac{\partial\bar{u}}{\partial y}+\frac{\partial\bar{v}}{\partial x}\right)-\sin 2\varphi\left(\frac{\partial\bar{u}}{\partial x}-\frac{\partial\bar{v}}{\partial y}\right)\right]=0,$$
$$\frac{d}{dt}\equiv\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}+\bar{v}\frac{\partial}{\partial y},$$

where σ_{11}, σ_{22}, σ_{12} are stress tensor components; $\bar{u}$, $\bar{v}$, velocity vector components of the displacement of points of the medium; k_1, k_2, k_3, tension (compression) coefficients of points of the medium; φ, angle of rotation of the coordinate system relative to the principal axes; and ρ, density of the medium.

We shall give the equation of state of the medium in the following form:

$$E(k_1, k_2, k_3) = K_0(\delta^n-1)^2/(2n^2)+2b_0^2\delta^m D, \tag{2.2}$$

where c_0, b_0 are the longitudinal and transverse speeds of sound in the medium, $K_0=c_0^2-(4/3)b_0^2$, and n and m are constants of the equation of state;

$$\delta = 1/(k_1k_2k_3);\quad D=\left(d_1^2+d_2^2+d_3^2\right)/2,$$
$$d_i=\ln\left(k_i/\sqrt[3]{k_1k_2k_3}\right)\quad (i=1,2,3).$$

The stress tensor components σ_{11}, σ_{22}, σ_{12} are determined in terms of the stresses σ_1, σ_2, and the angle φ by means of the formulas

$$\sigma_{11}=\sigma_1\cos^2\varphi+\sigma_2\sin^2\varphi,\quad \sigma_{22}=\sigma_1\sin^2\varphi+\sigma_2\cos^2\varphi,$$
$$\sigma_{12}=(\sigma_1-\sigma_2)\sin\varphi\cos\varphi.$$

The principal stresses are related to the equation of state by Murnaghan's formulas [3]

$$\sigma_i=\rho k_i E_i,\ E_i=\partial E/\partial k_i\ (i=1,2,3), \tag{2.3}$$

which, for the specific equation of state (2.2), take the form

$$\sigma_i=-\rho_0K_0\delta^{n+1}(\delta^n-1)/n-2\rho_0b_0^2m\delta^{m+1}D+2\rho_0b_0^2\delta^{m+1}d_i\qquad (i=1,2,3),$$

where ρ_0 is the density of the material under normal conditions.

At the initial instant the stress state is given in the (x, y) plane in the form

$$\sigma_{11}^0=\sigma_{33}^0=-k,\quad \sigma_{12}^0=0,\quad \bar{u}^0=\bar{v}^0=0,\quad \sigma_{22}^0=0\quad \text{in layer } S, \tag{2.4}$$

$$\sigma_{11}^0=\sigma_{33}^0=\sigma_{22}^0=\sigma_{12}^0=0,\ \bar{u}^0=\bar{v}^0=0\quad \text{in the half planes } P_1,\ P_2. \tag{2.5}$$

The tension (compression) coefficients of the medium k_1, k_2, k_3 can be reproduced by means of the stresses given. Since $\sigma_{12}^0=0$, then $\varphi^0=0$. It follows from (2.5) that $k_1^0=k_2^0=k_3^0=1$ in the half planes P_1 and P_2.

Let the initial stress state of the (x, y) plane be subjected to a small perturbation. We shall seek the solution for the perturbed state of system (2.1) in the form

$$k_1=k_1^0+\varepsilon\alpha(x,y),\quad k_2=k_2^0+\varepsilon\beta(x,y),\quad k_3=k_3^0,$$
$$\varphi=\varphi^0+\varepsilon\varphi^1(x,y),\quad \bar{u}=\bar{u}^0+\varepsilon u^1(x,y),\quad \bar{v}=\bar{v}^0+\varepsilon\bar{v}^1(x,y), \tag{2.6}$$

where $\varepsilon \ll 1$.

Substituting (2.6) into the last four equations in (2.1), and keeping only first order terms in ε in the relationships obtained, we obtain expressions for the functions α, β, φ^1:

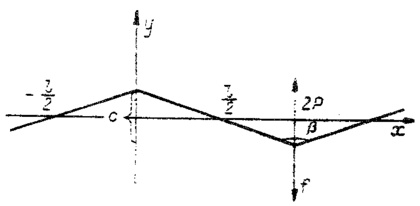

Fig. 1

$$\alpha = k_1^0 \frac{\partial u}{\partial x}, \quad \beta = k_2^0 \frac{\partial v}{\partial y}, \quad \varphi^1 = \frac{k_2^{02}}{k_1^{02} - k_2^{02}} \frac{\partial u}{\partial y} + \frac{k_1^{02}}{k_1^{02} - k_2^{02}} \frac{\partial v}{\partial x}, \tag{2.7}$$

where u, v are components of displacements of points of the medium from the initial to the perturbed state.

Let us use the relationships (2.7), Murnaghan's formulas (2.3), representation (2.6), and let us convert the first two equations of the system (2.1) to the form

$$\begin{gathered}
k_1^{02} E_{11}^0 \frac{\partial^2 u}{\partial x^2} + k_1^0 k_2^0 \left(E_{12}^0 + \frac{k_2^0 E_1^0 - k_1^0 E_2^0}{k_1^{02} - k_2^{02}} \right) \frac{\partial^2 v}{\partial x \partial y} + k_2^{02} \frac{k_1^0 E_1^0 - k_2^0 E_2^0}{k_1^{02} - k_2^{02}} \frac{\partial^2 u}{\partial y^2} = 0, \\
k_1^{02} \frac{k_1^0 E_1^0 - k_2^0 E_2^0}{k_1^{02} - k_2^{02}} \frac{\partial^2 v}{\partial x^2} + k_1^0 k_2^0 \left(E_{12}^0 + \frac{k_2^0 E_1^0 - k_1^0 E_2^0}{k_1^{02} - k_2^{02}} \right) \frac{\partial^2 u}{\partial x \partial y} + k_2^{02} E_{22}^0 \frac{\partial^2 v}{\partial y^2} = 0,
\end{gathered} \tag{2.8}$$

where $E_{ij} = \partial^2 E / \partial k_i \partial k_j$ $(i, j = 1, 2)$ and the superscript zero indicates that the quantities refer to the initial state.

Let us use the equation of state (2.2) and let us evaluate the coefficients of system (2.8):

$$\begin{gathered}
k_1^2 E_{11} = -\{-K_0 \delta^n (\delta^n - 1)/n - 2b_0^2 m \delta^m D + 2b_0^2 \delta^m d_1\} \\
+ \{K_0 (2\delta^{2n} - \delta^n) + 2b_0^2 m^2 \delta^m D - 4mb_0^2 \delta^m d_1 + 4b_0^2 \delta^m / 3\}, \\
k_1 k_2 \left(E_{12} + \frac{k_2 E_1 - k_1 E_2}{k_1^2 - k_2^2} \right) = K_0 (2\delta^{2n} - \delta^n) + 2b_0^2 m^2 \delta^m D - 2b_0^2 m \delta^m (d_1 + d_2) \\
- 2b_0^2 \delta^m / 3 + K_0 \delta^n (\delta^n - 1)/n + 2b_0^2 \delta^m m D + 2b_0^2 \delta^m \frac{k_2^2 d_1 - k_1^2 d_2}{k_1^2 - k_2^2}, \\
\frac{k_1 E_1 - k_2 E_2}{k_1^2 - k_2^2} = \frac{2b_0^2 \delta^m (\ln k_1 - \ln k_2)}{k_1^2 - k_2^2}, \\
k_2^2 E_{22} = -\{-K_0 \delta^n (\delta^n - 1)/n - 2b_0^2 m \delta^m D + 2b_0^2 \delta^m d_2\} + \{K_0 (2\delta^{2n} - \delta^n) + 2b_0^2 m^2 \delta^m D - 4b_0^2 m \delta^m d_2 + 4b_0^2 \delta^m / 3\}.
\end{gathered} \tag{2.9}$$

Taking into account that $E_2^0 = 0$ for the layer S, we write (2.8) in the form

$$\begin{gathered}
k_1^{02} E_{11}^0 \frac{\partial^2 u}{\partial x^2} + k_1^0 k_2^0 \left(E_{12}^0 + \frac{k_2^0 E_1^0}{k_1^{02} - k_2^{02}} \right) \frac{\partial^2 v}{\partial x \partial y} + k_2^{02} \frac{k_1^0 E_1^0}{k_1^{02} - k_2^{02}} \frac{\partial^2 u}{\partial y^2} = 0, \\
k_1^{02} \frac{k_1^0 E_1^0}{k_1^{02} - k_2^{02}} \frac{\partial^2 v}{\partial x^2} + k_1^0 k_2^0 \left(E_{12}^0 + \frac{k_2^0 E_1^0}{k_1^{02} - k_2^{02}} \right) \frac{\partial^2 u}{\partial x \partial y} + k_2^{02} E_{22} \frac{\partial^2 v}{\partial y^2} = 0.
\end{gathered} \tag{2.10}$$

The coefficients of system (2.10) are determined from (2.9) by substituting k_1^0, k_2^0, k_3^0, which are defined by (2.4). Since $k_1^0 = k_2^0 = k_3^0 = 1$ in the half planes P_1 and P_2, system (2.8) consequently has the following form in this case:

$$\begin{gathered}
\frac{2(1-\sigma)}{1-2\sigma} \frac{\partial^2 u}{\partial x^2} + \frac{1}{1-2\sigma} \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0, \\
\frac{\partial^2 v}{\partial x^2} + \frac{1}{1-2\sigma} \frac{\partial^2 u}{\partial x \partial y} + \frac{2(1-\sigma)}{1-2\sigma} \frac{\partial^2 v}{\partial y^2} = 0,
\end{gathered} \tag{2.11}$$

where σ is Poisson's ratio of the half plane material:

$$\sigma = (3K_0 - 2b_0^2)/(2(3K_0 + b_0^2)).$$

In this case the stresses will be determined in the displacements by the equalities

$$\begin{gathered}
\sigma_{11} = \frac{E}{(1+\sigma)(1-2\sigma)} \left[(1-\sigma) \frac{\partial u}{\partial x} + \sigma \frac{\partial v}{\partial y} \right], \\
\sigma_{12} = \frac{E}{2(1+\sigma)} \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right], \\
\sigma_{22} = \frac{E}{(1+\sigma)(1-2\sigma)} \left[\sigma \frac{\partial u}{\partial x} + (1-\sigma) \frac{\partial v}{\partial y} \right].
\end{gathered}$$

where E is Young's modulus of the half plane material.

$$E = 9\rho_0 K_0 b_0^2 / (3K_0 + b_0^2).$$

3. We shall seek a solution of the form

$$u(x, y) = iU(y) e^{i\omega x}, \quad v(x, y) = V(y) e^{i\omega x} \tag{3.1}$$

for system (2.10) describing the stress state in the layer S and the system (2.11) describing the perturbed state in the half planes P_1 and P_2.

Substituting (3.1) into (2.11) yields a system of ordinary differential equations

$$\begin{aligned} &\frac{d^2U}{\partial y^2} + \frac{\omega}{1-2\sigma}\frac{dV}{dy} - \omega^2\frac{2(1-\sigma)}{1-2\sigma}U(y) = 0, \\ &\frac{2(1-\sigma)}{1-2\sigma}\frac{d^2V}{\partial y^2} - \frac{\omega}{1-2\sigma}\frac{\partial U}{\partial y} - \omega^2 V(y) = 0; \end{aligned} \tag{3.2}$$

Let us determine the reaction of the elastic half planes P_1 and P_2 to a given bending of their boundaries. Let the following boundary conditions be satisfied: $\sigma_{12}(x, -h) = 0$, $v(x, -h) = B_0 e^{i\omega x}$, $\sigma_{12}(x, h) = 0$, $v(x, h) = A_0 e^{i\omega x}$. The boundary conditions for system (3.2) have the form

$$V(-h) = B_0, \quad \left[\frac{dU}{dy} + \omega V\right]_{y=-h} = 0; \tag{3.3}$$

$$V(h) = A_0, \quad \left[\frac{dU}{dy} + \omega V\right]_{y=h} = 0. \tag{3.4}$$

The solution of system (3.2) in the half plane P_1, which decreases at infinity and takes on the given boundary conditions (3.3) for $y=-h$, is given by the formulas

$$U(y) = \frac{B_0 e^{\omega(y+h)}}{2(1-\sigma)}[(2\sigma-1) - \omega(y+h)], \quad V(y) = \frac{B_0 e^{\omega(y+h)}}{2(1-\sigma)}[2(1-\sigma) - \omega(y+h)].$$

Correspondingly, the solution of system (3.2) in the half plane P_2, which decreases at infinity and takes on the given boundary conditions (3.4) for $y=h$, is given by the formulas

$$U(y) = -\frac{A_0 e^{\omega(h-y)}}{2(1-\sigma)}[2\sigma - 1 + \omega(y-h)], \quad V(y) = \frac{A_0 e^{\omega(h-y)}}{2(1-\sigma)}[2 - 2\sigma + \omega(y-h)].$$

Then σ_{22}, the stress tensor component on the half plane boundaries $y=-h$, $y = h$, is reproduced by means of the displacements obtained:

$$\sigma_{22}(x, -h) = \frac{E\omega}{2(1-\sigma^2)}B_0 e^{i\omega x}; \tag{3.5}$$

$$\sigma_{22}(x, h) = -\frac{E\omega}{2(1-\sigma^2)}A_0 e^{i\omega x}. \tag{3.6}$$

Now we seek a solution taking the given values (3.5) and (3.6) on the boundaries $y=-h$ and $y=h$ and $\sigma_{12}(x, -h) = 0$, $\sigma_{12}(x, h) = 0$ and having the form (3.1) for the system of equations (2.10) in the layer S. Let us introduce the notation

$$c_1 = c_2 = k_1^0 k_2^0\left(E_{12}^0 + \frac{k_2^0 E_1^0}{k_1^{02} - k_2^{02}}\right); \quad c_3 = k_1^{02}\frac{k_1^0 E_1^0}{k_1^{02} - k_2^{02}};$$

$$c_4 = -k_1^{02}E_{11}^0; \quad c_5 = -k_2^{02}E_{22}^0; \quad c_6 = k_2^{02}\frac{k_1^0 E_1^0}{k_1^{02} - k_2^{02}}.$$

Substituting (3.1) into system (2.10), we obtain a system of ordinary differential equations

$$\begin{aligned} &c_6\frac{d^2U}{dy^2} + c_1\omega\frac{dV}{dy} + c_4\omega^2 U = 0, \\ &c_5\frac{d^2V}{dy^2} + c_2\omega\frac{dU}{dy} + c_3\omega^2 V = 0. \end{aligned} \tag{3.7}$$

We write the solution of system (3.7) in the form

$$U(y) = m_1[a\cos\nu\omega y - b\sin\nu\omega y]e^{\mu\omega y} + m_2\{b\cos\nu\omega y +$$

$$+ a\sin\nu\omega y]e^{\mu\omega y} + m_3[-a\cos\nu\omega y - b\sin\nu\omega y]e^{-\mu\omega y} + m_4[-b\cos\nu\omega y + a\sin\nu\omega y]e^{-\mu\omega y},$$

$$V(y) = m_1\cos\nu\omega y e^{\mu\omega y} + m_2\sin\nu\omega y e^{\mu\omega y} + m_3\cos\nu\omega y e^{-\mu\omega y} - m_4\sin\nu\omega y e^{-\mu\omega y},$$

where

$$\mu = \sqrt{-(c_3c_6 + c_4c_5 - c_1c_2)/(4c_5c_6) + \sqrt{c_3c_4/(c_5c_6)}/2};$$

$$\nu = \sqrt{(c_3c_6 + c_4c_5 - c_1c_2)/(4c_5c_6) + \sqrt{c_3c_4/(c_5c_6)}/2};$$

$$a = -c_5\mu/c_1 - c_3\mu/(c_1(\mu^2 + \nu^2)); \quad b = -c_5\nu/c_1 + c_3\nu/(c_1(\mu^2 + \nu^2));$$

m_1, m_2, m_3, m_4 are arbitrary constants;

$$B_0 = V(-h) = m_1\cos\nu\omega h e^{-\mu\omega h} - m_2\sin\nu\omega h e^{-\mu\omega h} + m_3\cos\nu\omega h e^{\mu\omega h} + m_4\sin\nu\omega h e^{\mu\omega h};$$

$$A_0 = V(h) = m_1\cos\nu\omega h e^{\mu\omega h} + m_2\sin\nu\omega h e^{\mu\omega h} + m_3\cos\nu\omega h e^{-\mu\omega h} - m_4\sin\nu\omega h e^{-\mu\omega h}.$$

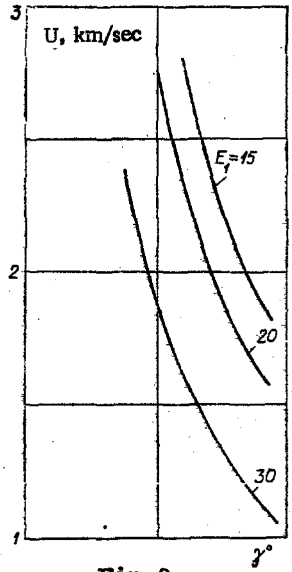

Fig. 2

The conditions on the boundary of the layer S yield the following algebraic system of equations in the constants m_1, m_2, m_3, m_4:

$$m_1A_{11} + m_2A_{12} + m_3A_{13} + m_4A_{14} = 0, \quad m_1A_{13} + m_2A_{14} + m_3A_{11}$$
$$+ m_4A_{12} = 0, \quad m_1B_{11} + m_2B_{12} + m_3B_{13} + m_4B_{14} = 0, \quad m_1B_{13} + m_2B_{14} + m_3B_{11} + m_4B_{12} = 0,$$

where

$$A_{11} = [(-a_{11} + R)\cos \nu\omega h + a_{12}\sin \nu\omega h]e^{\mu\omega h};$$
$$A_{12} = [-a_{12}\cos \nu\omega h + (-a_{11} + R)\sin \nu\omega h]e^{\mu\omega h};$$
$$A_{13} = [(a_{11} + R)\cos \nu\omega h + a_{12}\sin \nu\omega h]e^{-\mu\omega h};$$
$$A_{14} = [a_{12}\cos \nu\omega h - (a_{11} + R)\sin \nu\omega h]e^{-\mu\omega h};$$
$$B_{11} = (b_{11}\cos \nu\omega h - b_{12}\sin \nu\omega h)e^{\mu\omega h}; \quad B_{12} = (b_{12}\cos \nu\omega h$$
$$+ b_{11}\sin \nu\omega h)e^{\mu\omega h}; \quad B_{13} = (b_{11}\cos \nu\omega h + b_{12}\sin \nu\omega h)e^{-\mu\omega h};$$
$$B_{14} = (b_{12}\cos \nu\omega h - b_{11}\sin \nu\omega h)e^{-\mu\omega h};$$
$$a_{11} = b_1k_1^0a - b_2k_2^0\mu; \quad a_{12} = b_1k_1^0b - b_2k_2^0\nu;$$
$$b_1 = \rho^0k_2^0E_{12}; \quad b_2 = \rho^0k_2^0E_{22}^0;$$
$$b_{11} = (\mu a - b\nu)\frac{k_2^{02}}{k_1^{02} - k_2^{02}} + \frac{k_1^{02}}{k_1^{02} - k_2^{02}};$$
$$b_{12} = \frac{k_2^{02}}{k_1^{02} - k_2^{02}}(\mu b + a\nu); \quad R = \frac{E}{2(1 - \sigma^2)}.$$

A linear homogeneous system of algebraic equations has a solution different from zero if the determinant of this system equals zero; i.e.,

$$[-2b_{12}R\cos 2\nu\omega h - 2(b_{11}a_{11} + a_{12}b_{12})\sin 2\nu\omega h - b_{12}R(e^{-2\mu\omega h}$$
$$+ e^{2\mu\omega h}) + (a_{12}b_{11} - a_{11}b_{12})(e^{-2\mu\omega h} - e^{2\mu\omega h})][-2b_{12}R\cos 2\nu\omega h - 2$$
$$\times(b_{11}a_{11} + a_{12}b_{12})\sin 2\nu\omega h + b_{12}R(e^{-2\mu\omega h} + e^{2\mu\omega h}) - (a_{12}b_{11} - a_{11}b_{12})(e^{-2\mu\omega h} - e^{2\mu\omega h})] = 0. \quad (3.8)$$

We seek the roots of (3.8) numerically. The roots of (3.8) will be complex, of the form $\omega = \theta + i\psi$ for those values of the compressive stresses in the layer S which are given by (1.1). If $\omega = \theta + i\psi$ is a root, then $\bar{\omega} = \theta - i\psi$ is also a root of (3.8). Let us take the root which has the lesser positive imaginary part. This root will yield the solution which decreases least as $x \to \infty$ $(x > 0)$. For instance, for iron under the compressions $\sigma_{11}^0 = -19$ kbar, $\theta = 2.89$, $\psi = 0.826$. The length λ decreasing least for $x \to \infty$ of the wave by which the layer is bent is hence determined by the formula $\lambda = 2\pi/\theta$. Let us consider plastic hinges to be formed at the vertices of the half planes because of such a bending of the layer S. The layer will henceforth be considered as a "pin-joint system." The length l of the "rods" equals half the wavelength; i.e., $l = \pi/\theta$.

4. Let us calculate the reactive force of the half plane P_1 to the break in its boundary (Fig. 1). The equilibrium stress state in the half plane is described by system (2.11). We give the vertical displacement of the boundary of the half plane P_1 as follows:

$$v(x, -h) = \frac{4C}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}\cos[(2k-1)\theta x], \tag{4.1}$$

where C is the amplitude of the bending.

Let us seek the solution of (2.11) satisfying condition (4.1) and $\sigma_{12}(x, -h) = 0$ in the form

$$\begin{gathered} u(x, y) = \sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}U_{2k-1}(y)\sin[(2k-1)\theta x], \\ v(x, y) = \sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}V_{k-1}(y)\cos[(2k-1)\theta x]. \end{gathered} \tag{4.2}$$

Then a solution of the form (4.2) in which

$$U_{2k-1}(y) = -\left[r_{2k-1} - \frac{4\sigma-3}{(2k-1)\theta}d_{2k-1} + d_{2k-1}(y+h)\right]e^{(2k-1)\theta(y+h)},$$

$$V_{2k-1}(y) = [r_{2k-1} + d_{2k-1}(y+h)]\,e^{(2k-1)\theta(y+h)},$$

where

$$r_{2k-1} = \frac{4C}{\pi^2};\quad d_{2k-1} = -\frac{4C}{\pi^2}\frac{(2k-1)\theta}{2(1-\sigma)},$$

will be the desired solution. The reactive force P of the half plane P_1 to a break in the boundary, which is lumped at the vertex of the break and has a vertical direction (see Fig. 1), is determined in the form

$$P = \int_{-l/2}^{l/2}\sigma_{22}(x, -h)\,dx = \frac{4C}{\pi^2}\frac{E}{(1-\sigma^2)}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)^2} = \frac{4C}{\pi^2}\frac{E}{(1-\sigma^2)}G, \tag{4.3}$$

where the Catalan constant is G = 0.915965594....

The reaction of the two half planes to bending of the foundations is 2P. It is natural to assume that there are plastic deformation zones located in the half planes in the neighborhoods of the break in their boundaries. In this connection, the quantity $E/(1-\sigma^2)$ in (4.3) is replaced by $\tilde{E}/(1-\tilde{\sigma}^2) = E_0$, the characteristic modulus in the plastic deformation zone. For iron $\rho_0 = 7.84$ g/cm^3, $b_0 = 2.8$ km/sec, $c_0 = 5.7$ km/sec, n = 0.63, m = 2.7, $\sigma_S = 20$ kbar, and the quantity θ for compressions k close to σ_S lies within the limits $1.43 \leq \theta h \leq 1.45$. The height of the layer h is taken from [1]. For the layer to be broken, it is necessary that the force be greater than 2P; i.e.,

$$U > \frac{2E_0 G}{\theta h}\frac{3\tau_s}{4\rho_0 H}\left(\frac{c_0^2-b_0^2}{c_0^2-2b_0^2}\right)\frac{1}{\sin(\gamma/2)}. \tag{4.4}$$

Let us use (4.4) and let us construct a curve separating the domain of those values of U and γ at which a break in the layer is possible in the (U, γ) plane. The points U and γ which lie above the curves shown in Fig. 2 correspond to values of the velocities of the contact point and the collision angles for which a break in the layer is possible. Curves are constructed for iron for different values of $E_1 = E(1-\tilde{\sigma}^2)/\tilde{E}(1-\sigma^2)$ ($\tau_S = 1$ μsec).

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